**Modeling the Impact of Global Warming on Ecosystem Dynamics: A Compartmental Approach to Sustainability**

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1. **Generalized Sustainability Socio-Ecological Model (GSSEM)**

The socio-ecological model is described in this document.

***A.1. Full model description***

The model is a conceptual representation of a macroeconomic society within a simplified ecological food web, which was built in mass compartments that represents particular ecological or social components (Rodriguez-Gonzalez et al., 2018). A total of 14 compartments conform the model structure, three of which are resource pools: *RP* represents all biological resources (water, nutrients, etc.), *IRP* represents biological resources that are not primary accessible to all compartments due to human activity, and *ES* represents finite non-renewable energetic resources (oil, natural gas, etc.); six compartments are wild species with no economic value: *P2* and *P3* are primary producers, *H2* and *H3* are herbivores, and *C1* and *C2* are carnivores. *P2*, *H2* and *C1* interact whit the human sector through fence and grazing labors; four compartments are generic representations of industries in a society: *P1* represents the agricultural sector, *H1* is the livestock industry, *EP* is the energy production sector, and *IS* includes all manufacturing and services industries; finally, human sector is represented by HH compartment.

Initially, the model uses Equation (A.1) to estimate the per capita wage *Wi* for each population *i*, according to both the supply-demand gap of the product *IS* and the available labor; then, a global per capita wage is obtained by weighting the value of each population, as shown in Equation (A.1).

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|  | (A.1) |
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*ISdef* and *IStg* are the deficit and the inventory objectives of the *IS* compartment, respectively; *NHH* is the population size; *λ* and *θ* are the yields of *RP* and *P1,* respectively; *awi*, *cwi* y *dwi* are constant parameters; *α1* and *α2* are weight factors dependent on the total population fraction.

The economic sectors (*P1, H1, IS* y *EP*) define the production (*P1prod*, *H1prod*, *ISprod* y *EPprod*) and the price of their products (*P1p*, *H1p*, *ISp* y *EPp*) before knowing their demands from *HH*. Equations (A.2)-(A.7) represent the relation between the supply-demand gap and the prices of the products; if the supply-demand gap of *P1*, *H1* and *SI* increases, both prices and products will decrease. Further, changes in wages will also impact directly the prices of each product and their productions. For energy, there is no inventory; so, its price depends only on the energy resource reserves *ERP* and its production of total energy demand in the system, as it can be seen in Equations (A.8) and (A.9).

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|  | (A.2) |
|  | (A.3) |
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|  | (A.6) |
|  | (A.7) |
|  | (A.8) |
|  | (A.9) |

*P1def* and *H1def* are the inventory deficits of *P1* and *H1*; *P1tg* and *H1tg* are their inventory targets; *ISdem* is the total demand for *IS*. *EPHH* is the total demand for energy by *HH*. *EEIS* is the demand for energy by *IS*; *a, b* and *c* are constant parameters of the system.

Once the prices and productions are determined, all the demands of the system are calculated. The per capita demand for the products of each of the four economic activities by each human compartment *i* are interrelated and compete among each other, depending on the prices and demands of all of them. The actual values are determined by Equations (A.10) - (A.13); then, global demands are calculated by Equations (A.14) - (A.17).

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|  | (A.10) |
|  | (A.11) |
|  | (A.12) |
|  | (A.13) |
|  | (A.14) |
|  | (A.15) |
|  | (A.16) |
|  | (A.17) |

where *dki, kki, lki, nki, oki* and *zki* are constant parameters of the model for the variable *k* of the population *i*. By multiplying these global per capita demands by the total population *NHH*, the total demands of products are obtained (Equations (A.18)-( A.21)).

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|  | (A.18) |
|  | (A.19) |
|  | (A.20) |
|  | (A.21) |

The demand of the agricultural sector by the livestock sector *P1H1* is a function of wages. The price of P1 and the supply-demand gap is presented in Equation (A.22), where *dP1H1*, *eP1H1*, *f P1H1* and *gP1H1* are constant parameters.

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|  | (A.22) |

The industrial sector *IS* requires the mass consumption of *P1* and *RP*, in addition to the energy consumption of *EP*; Equations (A.23) - (A.25) estimate the demand from *IS* of each of them.

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|  | (A.23) |
|  | (A.24) |
|  | (A.25) |

The following step is to determine the other flows in which the human interferes directly. Access by the human sector to plants owned by the *H1P2* government is regulated by the latter, and it is equal to a constant factor *ǩ*, as presented in Equation (A.26).

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|  | (A.26) |

Equations (A.27) and (A.28) represent the consumption of *P1* and *H1* by the wild species *H2* and *C1*, which are limited by the natural growth and death of the former, as well as by their production; the production act as a fence that allows that wild species have access only to leftovers.

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|  | (A.27) |
|  | (A.28) |

*RPP1* is the growth of *P1* due to the consumption of *RP*; *P1H1* and *P2H1* are the growth of *H1* due to consumption of *P1* and *P2*; *P1RP* and *H1RP* are the mass flows towards *RP* due to the death of those compartments*.*

All that human beings consume come from *IS*, as well as all of the energy produced in the system; they then generate mass residues that are discarded and passed directly to the source of inaccessible resources *IRP*. These mass amounts are determined through Equations (A.29) and (A.30), by using their corresponding productivities (unit of mass per unit of product).

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|  | (A.29) |
|  | (A.30) |

The next step is the determination of all natural mass flows. Although *IRP* is a product of the waste from human activity, making the mass of the compartment inaccessible to humans, this mass is gradually reincorporated into the system through the primary producers *P2* and *P3*. This is represented by Equations (31) and (32).

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|  | (A.31) |
|  | (A.32) |

where *rP2* and *rP3* are recycling factors; *IRPRP* and *RPIRP* are constant flows between the main and inaccessible resource sources.

The model is based on Lotka-Volterra equations; then, each of the flows between compartments is a term of a multivariate prey-predator equation. Each flow is represented by a compound name indicating first the source compartment and then the compartment of destiny. There are two types of mass flows that are not determined by human action or by recycle: *i)* flows representing the growth of one compartment due to the consumption of another, which are determined by a bilinear product of both compartments and by a growth factor *gk*, as in Equations (A.33)-( A.41); and *ii)* mass flows due to the death of the compartments, estimated by the product of each compartment by its mortality rate *mk*, as in Equations (A.42)-( A.50).

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|  | (A.33) | |
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|  | (A.49) | |
|  | (A.50) | |
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*HHi is* the mass of the human compartment *i,* calculated by Equation (A.51).

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|  | (A.51) |

The following variables to be determined are the demographic variables. The per capita mass *pcmi* of each population *i* relate the amount of mass of the human compartment *HHi* and to the size of the population *NHHi*, that is not a mass variable, as presented in Equation (A.52).

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|  | (A.52) |

The per capita births *pcbi* for population *i* are determined by Equation (A.53), with a variable birth rate *apcbi*, the constant parameter *bcpbi*, the global per capita wages *W* and the variable *wp*. *wp* is calculated by weighting the total amount of each product consumed by *HH* with its respective sales price (Equation (A.54)).

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|  | (A.53) |
|  | (A.54) |

Although this model does not represent any particular human ecosystem, the main motivation of this work is to have a more realistic model that includes crucial and representative elements of real-world ecosystems. Due to their nature and complexity, ecosystems have inherent uncertainties that must be considered when performing a sustainability analysis in order to have more accurate results. Two crucial parameters in the dynamics of the model are human mortality and birth rate. These parameters are represented by using Stochastic Differential Equations (SDE) through Ito processes, as shown in Equations (A.55) and (A.56).

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|  | (A.55) |
|  | (A.56) |

Where *mHHij* and *apcbij* are uncertain dynamic variables for population *i* and integration time *j*, *Δt* is the integration step size, and *σmi* and *σbi* are the variance parameters; *εm* and*εb* are random values that have a unit normal distribution with zero mean and unit standard deviation.

Equations (A.57)-(A.74) are differential equations that represent the mass transfer dynamics of each of the compartments and their deficits. These equations are mass balances, where the rates of change are given by the difference between all in-out flows of each compartment.

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|  | (A.57) |
|  | (A.58) |
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|  | (A.69) |
|  | (A.70) |
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|  | (A.73) |
|  | (A.74) |

The number of humans in both populations *NHHi* is not a compartment because it has no mass, despite being closely related to *HHi*. However, its rate of change can be estimated in an analogous way through Equation (A.75). Its value depends on the increase due to births and on the decrease due to natural deaths and to a health factor. Equation (A.75) also includes an Economic Mobility Factor (EMF), which represents an interaction between both populations, as given by Equation (A.76):

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|  | (A.75) |
|  | (A.76) |

Each term is rounded to the nearest upper integer to avoid having a fraction of people. *pcmid* is an ideal per capita mass parameter; while *φi* is a weighting factor that relates human deaths to the difference between the actual per capita weight and the ideal of a human. EMF depends on the deviation of an ideal total income, the IEI index, and a flow direction factor *ψ*. *ψ* has a value of 1 for a population migrating to a level of higher income and -1 if it migrates to a lower level; *Wid* and *NHHid* are the ideal per capita salary and the ideal total population, respectively.

The model is delimited through a series of logical constraints. If *P1, H1, H2* or *C1* are equal to zero, their associated variables due to economic activity are also zero, as shown in Equations (A.77)-(A.80).

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|  | (A.77) |
|  | (A.78) |
|  | (A.79) |
|  | (A.80) |

If the source of inaccessible resources IRP is equal to zero, then the IRP compartment recycles flows to the primary producer P2 and P3 becomes zero (Equation (A.81)).

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|  | (A.81) |

Similarly, if the human compartment *HH* becomes zero, or the number of humans is below a minimum number, then, all of the flows due to human activity become zero and the access of both *H2* to *P1* and *C1* to *H1* are determined as terms of a Lotka-Volterra equation. See Equation (A.82).

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|  | (A.82) |

Equations (A.83) and (A.84) avoid negative compartments. They cannot be consumed if they are below the limit where they are no longer reproduced (*belowrep*). If the *i-th* compartment *yi* is below this limit, two situations can occur. If the flow due to the death of the compartment is greater than the sum of each of the j-th inflows, *finj*, then each flow of output *foutj* is equal to zero and the flow of mortality is equal to *finj*; if, on the other hand, the death stream is smaller, each of the output streams *foutj* is weighted so that the accumulation of the compartment, *yi*, is zero.

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|  | (A.83) |
|  | (A.84) |

Finally, once all internal variables are calculated, the model estimates the atmospheric GHG concentration in carbon monoxide equivalents () using Equation A.85. Each compartment has an emission factor () that represents its contribution to the CO2eq concentration per unit. Consequently, one can estimate, for each time step, the total contribution of human sector activity and the mitigation by the natural sector to that concentration. This procedure allows for monitoring the dynamics of CO2eq according to the behavior of the system.

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|  | (A.85) |

***A.2. Data sets used in Case Study 3.***

The data used for the simulations are provided in this appendix through Table A.1 to Table A.3.

Table A.1. Initial values of state variables

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| **State Variables** | **Initial value** | **State Variables** | **Initial value** |
|  | 0.127639522 |  | 1000 |
|  | 6.637479579 |  | 0 |
|  | 1.181396149 |  | 0 |
|  | 1.248945367 |  | 0 |
|  | 0.065892868 |  | 0 |
|  | 1.073417243 |  | 0 |
|  | 1.358944396 |  | 750 |
|  | 0.611883366 |  | 250 |
|  | 0.450700000 |  | 0.338025000 |
|  | 0.508187978 |  | 0.112675000 |
|  | 20.10894289 |  | 300 |
|  | 0.881746274 |  | 25 |
|  | 800 |  |  |

Table A.2. Values of System parameters

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| --- | --- | --- | --- | --- | --- |
| **Parameter** | **Value** | **Parameter** | **Value** | **Parameter** | **Value** |
| Demographic, social and technological | | | | | |
|  | 1000 |  | 2.66E-04 |  | 0.1 |
|  | 20 |  | 1.56E-03 |  | 0.676677233 |
|  | 10 |  | 2.0E-03 |  | 0.101991961 |
|  | 5 |  | 310 |  | 1 |
|  | 0.00543608 |  | 4.51E-05 |  | 0.2 |
|  | 2.34E-05 |  | 0.1 |  | 0.04 |
|  |  |  |  |  |  |
| Growing | | Economic | | Demand | |
|  | 0.003541127 |  | 0.43853 |  | 0.000191077 |
|  | 0.009933643 |  | 0.135718104 |  | 0.049912497 |
|  | 0.000778772 |  | 4.51E-06 |  | 0.81332 |
|  | 0.058687036 |  | 0.4968 |  | 2.9657 |
|  | 0.0168 |  | 0.67631 |  | 4.00E-08 |
|  | 0.125249403 |  | 0.12318 |  | 1.60E-07 |
|  | 0.366996266 |  | 1.4359 |  | 6.00E-08 |
|  | 0.052509103 |  | 0.001 |  | 0.00E+00 |
|  | 0.117534846 |  | 0.252716513 |  | 1.60E-07 |
|  | 0.079785 |  | 1.17 |  | 6.00E-08 |
|  | 0.19963 |  | 0.297210307 |  | 6.00E-08 |
|  | 0.021472781 |  | 0.001 |  | 6.00E-08 |
|  | 0.357331692 |  | 0.01 |  | 6.00E-08 |
|  |  |  | 0.4968 |  | 0 |
| Mortality | |  | 0.67631 |  | 6.00E-08 |
|  | 0.001018295 |  | 615.9 |  | 3.13E-05 |
|  | 0.197313146 |  | 0.050392 |  | 6.00E-08 |
|  | 0.186325524 |  | 0.149737492 |  | 6.00E-08 |
|  | 0.009838862 |  | 0.033805381 |  | 4.00E-08 |
|  | 0.0004 |  | 0.24182 |  | 2.00E-08 |
|  | 0.196123663 |  | 0.049912497 |  | 6.00E-08 |
|  | 0.092105574 |  | 0.26657 |  | 5.68E-05 |
|  | 0.171458886 |  | 0.3109 |  | 6.00E-08 |
|  | 0 |  | 0.0044 |  | 6.00E-08 |
|  | 0.49337505 |  | 0.3313 |  | 4.00E-08 |
|  |  |  | 0 |  | 2.00E-08 |
|  |  |  | 0.4 |  | 6.00E-08 |
|  |  |  | 0 |  | 5.68E-05 |

Table A.3. Global Warming related parameters.

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| --- | --- | --- | --- |
|  |  |  |  |
|  | 24.637 |  | 1.237E+3 |
|  | 1.785 |  | 1.050E+3 |
|  | 7.430E-3 |  | -1.692 |
|  | 2.882E+3 |  | -1.663 |
|  | 1.555E+3 |  | -0.462 |
|  | 25 |  | 0.22024 |